Bateman, A., Li, M., & McGeehan, J. P. (1992). Theoretical performance comparison between reference-based coherent BPSK and BCH coded differential BPSK. In Globecom 1992, Orlando. (Vol. 3, pp. 1791 - 1796). Institute of Electrical and Electronics Engineers (IEEE). 10.1109/GLOCOM.1992.276692

Link to published version (if available):
10.1109/GLOCOM.1992.276692

THEORETICAL PERFORMANCE COMPARISON BETWEEN REFERENCE-BASED COHERENT BPSK AND BCH CODED DIFFERENTIAL BPSK

Mu Li, J.P. McGeehan and A. Bateman

University of Bristol
Centre for Communications Research
Queens Building, University Walk
Bristol BS8 1TR, United Kingdom
Tel: +44 272 303726, Fax: +44 272 255265

ABSTRACT Both channel sounding and channel coding are well-known techniques for improving BER performance of data transmission over a narrow band mobile radio channel. The two techniques share a common ground with regard to the fact that both of them employ redundancy signals (reference signals for channel sounding and parity check bits for channel coding) to achieve the improved performance. In this paper, a comparison of theoretical PSK performance in BER is made between the reference-based coherent detection and the BCH coded differential detection under different fading conditions. Comparisons are also made between reference-based coherent detection and BCH coded decision-feedback DPSK.

1 Introduction

It is a common phenomenon that signals received by a mobile from a base station as it moves about undergo violent and rapid variations in both amplitude and phase. These variations introduce great distortion and interference to data transmission. Attempts to achieve coherent detection by tracking the carrier phase have failed catastrophically in multipath fading condition [1]. Consequently, non-coherent detections such as differential detection and frequency discrimination are widely used in mobile data communication. Although these non-coherent detections show reasonable performance in tamed multipath conditions, they generally exhibit an unacceptably high error floor at high fading rate. In practical systems, channel coding schemes have to be applied to maintain the system performance, but with associated throughput penalty.

For years, reference-based channel sounding techniques have been explored for application to coherent data systems[2][3]. A reference tone (or reference symbols) is transmitted along with the data to provide the local reference signal, based upon the fact that a fading channel can be measured by transmitting a reference signal that is situated in its coherent bandwidth. Using the measured channel information to compensate the faded data signal, the channel-induced random phase can be eliminated. Consequently coherent detection, which can not be realized otherwise, can be achieved. However, it is also clear that an extra amount of power and bandwidth needs to be paid to accommodate the reference signal. For a bandwidth-limited system, system throughput has to be reduced to accommodate the redundant information.

Reference-based channel sounding can essentially be regarded as a modulation scheme. However, it shares a common ground with channel coding which is that both channel coding schemes and reference-based systems improve the performance by adding some redundancy such as reference signals in reference-based systems and parity check bits in channel coding schemes. Clearly, their mechanisms for performance improvement are different. For reference-based systems, reference signals provide carrier reference which is used for coherent detection. Systems employing channel coding improve their performance by error correction. With the same amount of redundancy used for both systems, which system can provide a better performance? This is not a straight forward question. In this paper, this issue is investigated by comparing BCH coded differential BPSK and reference-based coherent BPSK.

2 Performance of reference-based coherent BPSK

A received signal transmitted over a narrow band mobile radio channel in its complex baseband form can be ex-

0-7803-0608-2/92/$3.00 © 1992 IEEE

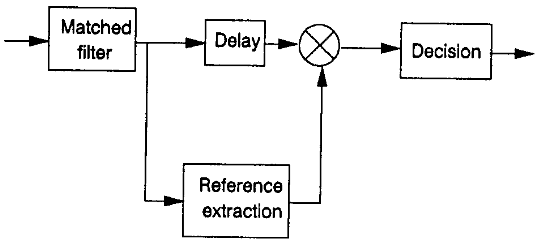

Figure 1: Coherent detection of reference-based system.

pressed by:

$$x(t) = \sqrt{\frac{E_s}{T}} g(t) \sum_{i=-\infty}^{\infty} c_i p(t - iT) + n(t) \qquad (1)$$

where c_i is a transmitted symbol, $p(t)$ is the symbol pulse with the normalization $\int_{-\infty}^{\infty} |p(t)|^2 = T$, E_s denotes the energy of one symbol pulse, T is the symbol period, $n(t)$ is the complex additive white Gaussian noise with double-sided spectral density N_o and $g(t)$ is the distortion term in a flat fading channel. As shown in Figure 1, after being filtered by a matched filter with impulse response $p^*(-t)$, sampling of the matched filter output at $t = kT$ and using the normalization give the follow discrete expression:

$$x_k = g_k c_k + n_k \qquad (2)$$

where n_k is the complex white Gaussian noise with its variance $\sigma_n^2 = E_s/N_o$. A narrow band filter for reference tone based systems or an interpolation filter for reference symbol based systems can be used to extract the reference signal. Assuming the extraction process is perfect, the reference signal can be expressed in general as:

$$y_k = g_k + u_k \qquad (3)$$

where u_k is the noise in reference signal duo to the filtered white Gaussian noise. Decision variable can be expressed in a general quadratic form:

$$D = x_k y_k^* + x_k^* y_k \qquad (4)$$

For BPSK, if transmitted symbol $c_k = 1$, an error will occur when $D < 0$. In an AWGN channel where $g_k = 1$ in equation (2) and equation (3), the error probability can be derived [4]:

$$\begin{aligned} P_e &= p[D < 0] \\ &= Q(a, b) - \frac{1}{2} I_o(ab) e^{-\frac{a^2+b^2}{2}} \qquad (5) \end{aligned}$$

where I_o is the zero order Bessel function, $Q(a, b)$ can be represented using series Bessel functions,

$$Q(a, b) = e^{-\frac{a^2+b^2}{2}} \sum_{n=0}^{\infty} \left(\frac{a}{b}\right)^n I_n(ab) \qquad (6)$$

and a, b are given by

$$\begin{aligned} a &= \sqrt{\gamma_1} - \sqrt{\gamma_2} \qquad (7) \\ b &= \sqrt{\gamma_1} + \sqrt{\gamma_2} \qquad (8) \end{aligned}$$

In a Rayleigh fading channel, bit error rate of reference-based BPSK can be derived [5]:

$$P_e = \frac{1}{2}\left(1 - \frac{1}{\sqrt{(\frac{1}{\gamma_1} + 1)(\frac{1}{\gamma_2} + 1)}}\right) \qquad (9)$$

where γ_1 and γ_2 are the SNR of the received signal x_k and the reference signal y_k respectively. Since reference signal is redundant signal which conveys no information, clearly $E_s \neq E_b$ where E_b is the energy per information bit. For a reference symbol based BPSK where the power of reference symbol is assumed to be the same as the power of information symbol in order to preserve the constant envelope of BPSK, it can be shown that the following relationships exist [6]:

$$\begin{aligned} \gamma_1 &= \frac{M-1}{M} \frac{E_b}{N_o} \qquad (10) \\ \gamma_2 &= \frac{M-1}{M} \frac{1}{MBT} \frac{E_b}{N_o} \qquad (11) \end{aligned}$$

where M is the number of symbols in between two successive reference symbols and B is the bandwidth of the interpolator. Substituting equation (10) and equation (11) into equation (5) and equation (9), BER performances of a reference-based BPSK in both AWGN channel and Rayleigh fading channel can be obtained.

3 BCH coded differential BPSK system

A coded differential BPSK system is used to compare with the reference-based coherent BPSK. As shown in Figure 2, BCH coding and interleaver are used in this coded differential BPSK system. The redundancy introduced by both BCH coding and reference signal insertion in these two systems are assumed the same so that the two systems have the same throughput. Under this condition, BER performances of the two systems are evaluated and compared.

3.1 BCH codes

The coding scheme chosen to compare with the reference signal insertion is BCH codes. BCH codes are a generalization of Hamming codes that allow multiple error correction. They are a powerful class of cyclic codes that provide a large selection of block lengths, code rates, alphabet sizes and error-correcting capability. For (n, k) block BCH codes with the error correction capability t, the probability that

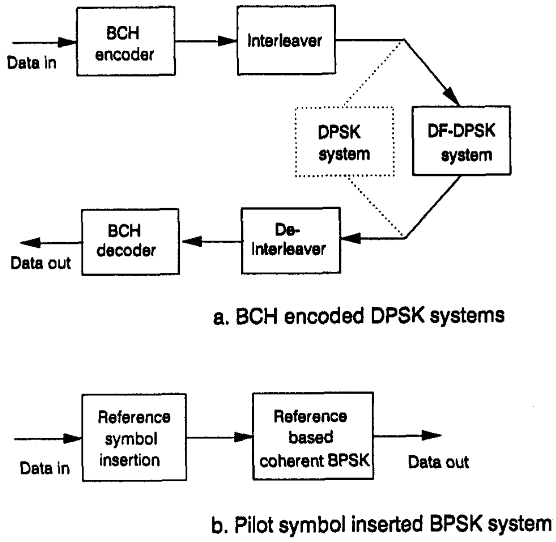

Figure 2: Illustrations of two systems to be compared.

the decoder commits an erroneous decoding and that the n-bit block is in error can be calculated by:

$$P_M \leq \sum_{j=t+1}^{n} \binom{n}{j} p^j (1-p)^{n-j} \qquad (12)$$

and decoded bit error probability can be expressed by the following approximation [4]:

$$P_b \simeq \frac{1}{n} \sum_{j=t+1}^{n} j \binom{n}{j} p^j (1-p)^{n-j} \qquad (13)$$

where p represents the channel error rate of the coded system.

3.2 Interleaving

In general, BCH codes considered are designed to combat random independent errors. That is, the channel in which it operates must be memoryless. A channel that has memory is the one that exhibits mutually dependent signal transmission impairments. An example of such channels is the fading channel, particularly when fading slowly compared to one symbol time. These time-correlated impairments result in statistical dependence among successive symbol transmissions. Therefore, the disturbances tend to cause errors that occur in bursts, instead of as random isolated events. Consequently, the bit errors in fading channel can not be characterized as single randomly distributed bit errors whose occurrence is independent from bit to bit. The result of this on coded signals is to cause degradation in error performance. One of the techniques to deal with this problem is the use of time diversity of interleaving. Interleaving the coded message before transmission and deinterleaving after receptions cause the bursts of errors to spread out in time and thus to be handled by the decoder as if they were random errors. Since in all practical cases, channel memory decreases with time separation, separating the symbols in time effectively transforms a channel with memory to a memoryless one, and therefore enables the random-error-correction codes to be useful in a burst error channel. As the interleaving period increases, the error performance can be improved in the sense that noise bursts are more dispersed. On the other hand, the delay due to interleaving and deinterleaving increases. Consequently, there is always a trade-off between error performance and interleaving delay.

3.3 Performance of coded differential BPSK

The performance of differential BPSK in an AWGN channel is given by

$$P_e = \frac{1}{2} e^{-\gamma} \qquad (14)$$

In a Rayleigh fading channel, the performance of differential BPSK can be approximated by [5]:

$$P_e = \frac{1}{2(\gamma+1)} + \frac{1 - J_o(2\pi f_d T)}{2} \qquad (15)$$

where the first term in equation (15) is the BER performance of the ideal differential BPSK in a Rayleigh fading channel and the second term in equation (15) is the irreducible error rate exhibited by the differential BPSK, f_d is the Doppler shift frequency, γ is E_c/N_o and E_c is the code symbol energy. For (n,k) coded BPSK, the energy per bit E_b is related to E_c by

$$E_c = \frac{k}{n} E_b \qquad (16)$$

Therefore, we have

$$\gamma = \frac{k}{n} \frac{E_b}{N_o} \qquad (17)$$

Substituting equation (15) into equation (13), the performance of coded BPSK in AWGN and Rayleigh fading channels can be obtained.

4 Result analysis and discussion

In this paper, reference symbol based BPSK is used for performance evaluation of reference-based coherent BPSK. Since both power and spectrum efficiencies of reference symbol based BPSK are the same as that of reference tone based BPSK [5], following analysis and discussion also apply to reference tone based systems.

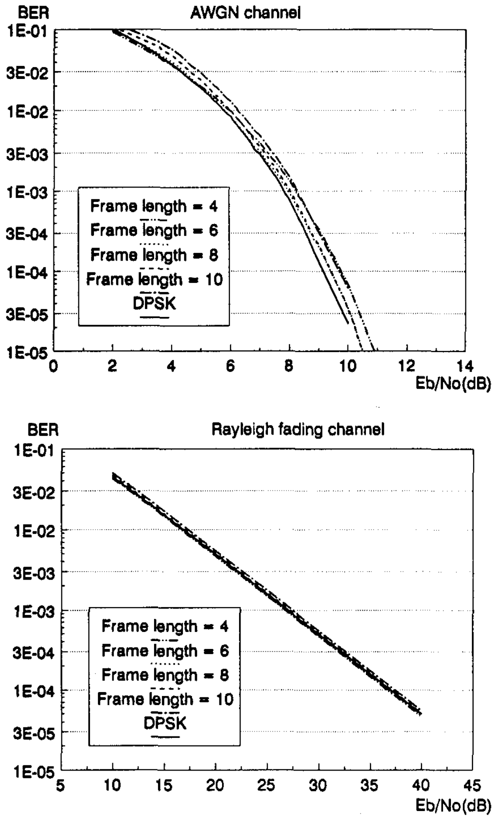

Figure 3: Performance of reference symbol based BPSK using different reference symbol insertion rate. Symbol rate = 2400 symbols/s, interpolator bandwidth = 240 Hz.

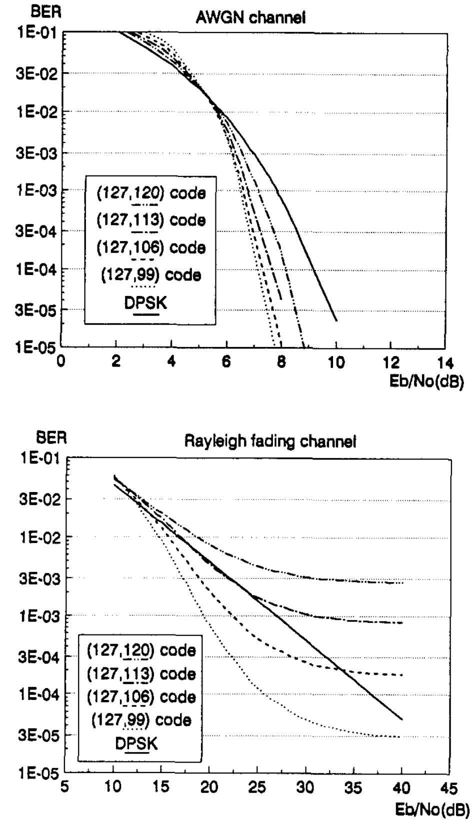

Figure 4: Performances of BCH coded DPSK with different coding rate. Symbol rate = 2400 symbols/s.

Figure 3 plots the performance of reference symbol based BPSK with frame length 4, 6, 8, 10 and Figure 4 plots the performance of BCH coded DPSK with coding (127, 99), (127,106), (127, 113) and (127,120) which have the system redundancy that are similar to 1 out of 4, 6, 8, 10 reference symbol insertion rate. It can be seen clearly that system performances of reference symbol based BPSK with different frame length have little variation, which means that there is little performance gain by adding extra redundancy into reference symbol based systems except that the requirement for interpolator design can be relaxed and fading combat capability can be raised (due to high channel sampling rate). However, the performance of coded BPSK system is improved greatly with the increase of coding redundancy. Figure 4 shows that the irreducible error floor is reduced by 100 fold from 3×10^{-3} to 3×10^{-5} as the code rate changes from 1/4 to 1/10. This is one major difference between the two systems. Given the reference symbol insertion rate and the fading signal interpolator bandwidth, the performance of reference symbol based system does not change with the fading rate as long as the fading rate is within the range that the system is designed to cope with. However, since the performance of DPSK is dependent upon channel fading rate, the coded DPSK with the same coding rate is expected to yield different performances for different channel fading rate. Figure 5 plots the performance of coded differential BPSK and reference symbol based BPSK for different Doppler shifts. Clearly, a coded differential BPSK system with the same overhead as a reference symbol based BPSK system could exhibit a better performance than reference symbol based system if the fading rate is below certain level. This is a very important observation. In reference symbol based systems, the reference symbol insertion rate and interpolator bandwidth are normally designed to cope with the worst case. In practice, however, the actual fading rate is usually below the designated level. Therefore, most of the time, the interpolator bandwidth is wider than necessary, which admits extra amount of white Gaussian noise. Even with an adaptive interpolator (bandwidth of the interpolator is adaptive to the fading rate), the performance of reference

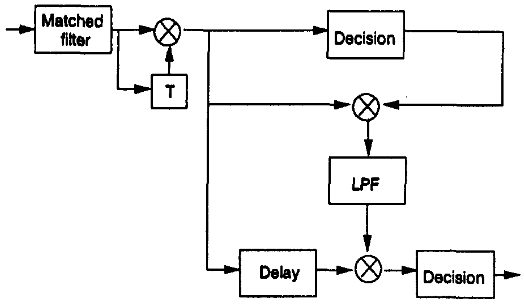

Figure 5: Block diagram of decision-feeback DPSK.

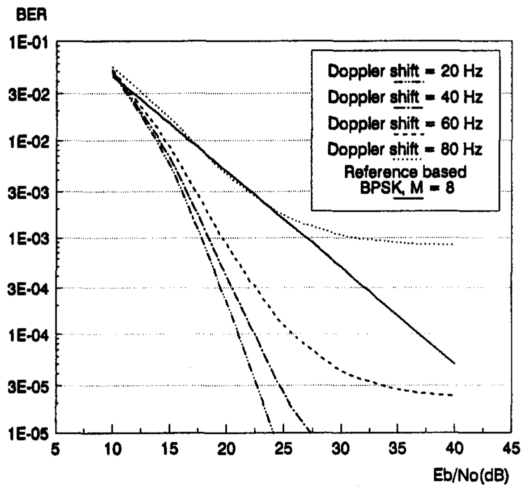

Figure 6: Performances of (127, 113) BCH coded DPSK and reference symbol based coherent BPSK in a Rayleigh fading channel with different fading rate. Symbol rate = 2400 symbols/s.

symbol based system is bounded by the performance of ideal coherent detection.

Since the performance of BCH coded DPSK is fading rate dependent, it shows certain adaptivity to the channel condition. In a slow fading channel, BCH coded DPSK exhibits a marginally better performance than reference symbol based systems with the same overhead used. In a fast fading channel, however, the performance improvement of BCH coded DPSK is severely limited by the irreducible error floor exhibited in DPSK. It can be seen in Figure 6 that although coded DPSK can suppress the error floor to some extent, error floor in the order 10^{-3} in a Rayleigh fading channel with Doppler shift 80Hz still exists. It is at this point that reference symbol based system shows its superiority. Clearly, the major obstacle to the performance improvement of coded differential BPSK lies with

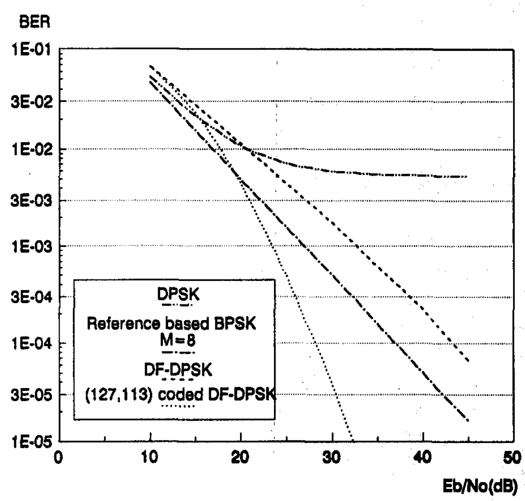

Figure 7: Performances of (127, 113) BCH coded decision-feedback DPSK and reference symbol based BPSK with symbol insertion rate of 1/8 in a Rayleigh fading channel. Symbol rate = 2400 symbols/s, Doppler shift = 80 Hz.

irreducible error floor. In [7][8], decision-feedback DPSK is proposed. Figure 5 illustrates the scheme. A main merit of decision-feedback DPSK is that error floor can be suppressed considerably by channel calibration without any overhead involved. Therefore, a combination of decision-feeback DPSK and BCH coding is expected to overcome the limitation suffered by the BCH coded DPSK in fast fading environment. Figure 7 plots the performance of uncoded differential BPSK and decision-feedback DPSK. It can be seen that decision feedback DPSK has a significant performance improvement over conventional DPSK. However, compared with reference symbol based BPSK, decision-feedback DPSK is still about 5dB worse. This is nonetheless a significant improvement considering that not any overhead information is required to achieve this. If decision-feedback DPSK is BCH coded with redundancy that is similar to reference symbol inserted BPSK, its performance is much better than the reference symbol based BPSK. This is borne out by Figure 7 which plots the performance of reference symbol based BPSK with frame length 8 and BCH coded decision feedback DPSK with the coding rate about 1/8. Since the situation considered in Figure 7 is near the worst case in which Doppler shift is assumed to be 80Hz, it can be concluded that the performance of coded decision-feedback DPSK is generally better than reference-based system with similar redundancy.

5 Conclusion

This paper presents the comparison between reference-based coherent BPSK and BCH coded differential BPSK under the condition that the same amount of redundancy is added to both systems. Results show that BCH coded differential BPSK can achieve better performances than reference-based coherent BPSK in slow fading channels. But in a fast fading channel, reference-based systems are marginally superior to coded differential BPSK which suffers from high irreducible error rate. In coded systems, performance is strongly dependent on the amount of redundancy used. Increase of redundancy will bring noticeable improvement in performance. For reference-based systems, increase of redundancy can increase the fading combat capability, but fails to gain performance advantage. Combining decision-feedback DPSK scheme with coding produces much better performance than reference-based systems in both slow and fast fading cases because of the superior performance of decision-feedback DPSK in fast fading channels.

References

[1] W.J. Weber, "Performance of phase-locked loops in the presence of fading communication channels," *IEEE Transactions on Communications*, Vol. COM-24, No. 5, May 1976.

[2] A. Bateman, J.P. McGeehan, "Data transmission over UHF fading mobile channel," *IEE Proceedings*, Vol.131, pt.F, No.4, July 1984.

[3] S. Sampei, T. Sunage, "Rayleigh fading compensating method for 16-QAM in digital land mobile radio channels," *Proceedings of IEEE 39th Vehicular Technology Conference*, San Francisco, May 1989.

[4] J. Proakis, *Digital Communication*, Second edition, McGraw-Hill, 1989.

[5] M. Li, "Data transmission over narrow band mobile radio channel using low overhead channel sounding," Ph.D thesis, University of Bristol, 1992.

[6] M. Li, A. Bateman J.P. McGeehan, "Evaluation of the optimum state for pilot symbol inserted 16-QAM," *Proceedings of IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, King's College, London, 1991.

[7] M. Li, A. Bateman, J.P. McGeehan, "Decision feedback channel estimation - a precursor for adaptive data transmission management," *41st IEEE Conference on Vehicular Technology*, 1991.

[8] M. Li, A. Bateman, J.P. McGeehan, "Analysis of decision-aided DPSK in the presence of multipath fading," *IEE Sixth International Conference on Mobile Radio and Personal Communications*, December, 1991.